A Class of Iterative FDE Techniques for Reduced-CP SC-Based Block Transmission

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Abstract

For conventional CP-assisted (Cyclic Prefix) block transmission systems, the CP length is selected according to the expected maximum delay spread. In this paper, an SDDC-aided (Soft Decision Directed Correction), iterative FDE (Frequency Domain Equalization) technique is presented for reduced-CP, SC-based (Single Carrier) block transmission systems using conventional frame structures. A more sophisticated Turbo SDDC-FDE technique is then proposed for improved performances in such systems, through a moderately increased complexity. The relations with some already known iterative FDE techniques are established, and a set of performance results is used to show the advantages of the SDDC-aided approach, namely the following: the fact that it can operate in an acceptable way even when, for the sake of implementation simplicity, no decoding operations are used to improve the iterative FDE process; the possibility of achieving the maximum power efficiency gain that a strong CP reduction allows.

1 Introduction

Conventional SC (Single Carrier) modulations have been shown to be suitable for CP-assisted (Cyclic Prefix) block transmission within broadband wireless communication systems [1]. With a CP long enough to cope with the maximum relative channel delay, a low-complexity FDE (Frequency-Domain Equalization) technique, involving simple FFT (Fast Fourier Transform) computations, can be employed to solve the severe ISI problem. In recent years, the possibility of achieving improved FDE performances in SC-based systems, under full-length CP conditions, was considered by several authors. One approach is a "turbo equalization in the frequency domain" (Turbo FDE) [2], [3], where the linear FDE and the decoding procedures (assuming coded data) are jointly performed, in an iterative way. An FDE approach with lower complexity is the Iterative Block Decision Feedback Equalization (IB-DFE), which does not use decoding within the iterative process: this approach, introduced in [4], was later extended and shown to be easily compatible with space diversity and MIMO systems [5], [6], as well as selected CP-assisted CDMA multiple access schemes [7]. Both FDE approaches can take advantage of state-of-the-art, low-cost, FFT-based technology.

Since the use of a full-length CP reduces both power and bandwidth efficiencies, the possibility of adopting a CP below the channel memory order, while keeping an essentially FFT-based implementation, deserves to be considered. In [8], a basic algorithm for a "decision-directed correction" (DDC) of the FDE inputs under reduced-CP conditions was shown to provide good performances, without significant error propagation, when using low-complexity, non-iterative receiver techniques, for specially designed SC-based frame structures. Selected time-domain computations, similar to those behind the DDC algorithm, were already proposed, within iterative algorithms which provide IBI suppression and CP reconstruction, for SC-based and OFDM-based block transmission systems [9], [10].

In this paper, in the reduced-CP context, we consider the use of a "Soft DDC" algorithm, in an iterative way, as an aid to either the conventional FDE/MMSE (Minimum Mean Squared Error) technique (Sec. 2) or improved, "turbo-type" FDE techniques as reported above (Sec. 3). Sec. 4 provides numerical performance results, and Sec. 5 concludes the paper.

2 Iterative SDDC-FDE Technique for Reduced-CP Transmission

For a length-L CIR (Channel Impulse Response), let us consider the transmission of length-N data blocks $\mathbf{s}(m) = [s_0(m), s_1(m), \ldots, s_{N-1}(m)]^T$ ($s_n(m)$ symbol coefficients taken from, e.g., a Quaternary Phase Shift Keying alphabet), with $N > L$. Whenever a length-L CP is appended to each data block, the length-N mth useful received block can be represented by $\mathbf{y}_{CP}(m) = \mathbf{H}\mathbf{s}(m) + \mathbf{n}(m)$, where $\mathbf{n}(m) = [n_0(m), \ldots, n_{N-1}(m)]^T$ is the mth received noise vector and $\mathbf{H}$ is the $N \times N$ circulant matrix which describes the channel effects. The entries of this square matrix, given by $h_{j,k} = h_{(j-k)\ mod\ N}$, are related to the length-L CIR ($h_n = 0$ for $n = L+1, \ldots, N-1$).

Let us consider the transmission of length-N blocks with a length-L_R CP ($0 \leq L_R < L, N + L_R \geq 2L$). Therefore, the initial $\Delta L = L - L_R$ samples of each received block will differ from the corresponding samples under full-length CP, unless

$$\Delta s_p(m) = s_p(m) - s_{p+L_R}(m-1) \qquad (1)$$

is equal to zero with $p = N - L, \ldots, N - L_R - 1$. The insufficient CP leads to some IBI and also to an imperfect circular convolution regarding the channel impact on the data block contents. Obviously, when using $\mathbf{y}(m)$ to denote the new length-N received block, $\mathbf{y}_{CP}(m) - \mathbf{y}(m)$ will depend on $\Delta s_p(m), p = N - L, \ldots, N - L_R - 1$. It can be shown that

$$\mathbf{y}_{CP}(m) - \mathbf{y}(m) = \mathbf{I}'_{\Delta L}\mathbf{H}\boldsymbol{\Delta}(m), \qquad (2)$$

with $\mathbf{I}'_{\Delta L}$ and $\boldsymbol{\Delta}(m)$ as follows:
$\mathbf{I}'_{\Delta L} = \mathbf{diag}[\underbrace{1,\ldots,1}_{\Delta L},\underbrace{0,\ldots,0}_{N-\Delta L}]$; $\boldsymbol{\Delta}(m) =$
$[\underbrace{0,\ldots,0}_{N-L},\underbrace{\Delta s_{N-L}(m),\ldots,\Delta s_{N-L_R-1}(m)}_{\Delta L},\underbrace{0,\ldots,0}_{L_R}]^T$.
If a perfect a priori knowledge of the ΔL pairs $(s_p(m), s_{p+L_R}(m-1))$ could be assumed, it should be possible, obviously, to "correct" the received vector $\mathbf{y}(m)$, by replacing it by the appropriate vector $\mathbf{y}_{CP}(m)$ prior to frequency-domain equalization. When an estimate $\widehat{\boldsymbol{\Delta}}(m)$ of $\boldsymbol{\Delta}(m)$ is available, a Decision-Directed Correction (DDC) of $\mathbf{y}(m)$ can be carried out to obtain a suitable approximation to $\mathbf{y}_{CP}(m)$:

$$\tilde{\mathbf{y}}_{CP}(m) = \mathbf{y}(m) + \mathbf{I}'_{\Delta L}\mathbf{H}\widehat{\boldsymbol{\Delta}}(m) =$$
$$= \mathbf{y}(m) + \mathbf{I}'_{\Delta L}\mathbf{F}^{-1}\mathbf{diag}[H_0,\ldots,H_{N-1}]\mathbf{F}\widehat{\boldsymbol{\Delta}}(m) \quad (3)$$

where $\mathbf{F}$ and $\mathbf{F}^{-1}$ denote a DFT matrix and an IDFT matrix, respectively, and $[H_0, H_1, \ldots, H_{N-1}]^T$ is the DFT of $[h_0, h_1, \ldots, h_{N-1}]^T$. This is the basis for the "DDC algorithm", as presented in [8].

In the following, we assume an SC-based block transmission, with length-N useful symbol blocks (corresponding to blocks of coded data) and a length-L_R cyclic prefix for every block. A continuous symbol stream is then transmitted, since the CP-assisted blocks are contiguous. This is a conventional frame for block transmission; however, we will consider that the CP length (L_R) can be chosen to be smaller than L.

In this context, a very simple iterative FDE technique using a "DDC aid" could easily be devised: for a given block m and a given FDE iteration $i \in \{1, 2, \ldots, I\}$, the length-$N$ time-domain input block $\tilde{\mathbf{y}}^{(i)}_{CP}(m)$ for FDE purposes could be obtained from the received vector $\mathbf{y}(m)$ simply by using the DDC algorithm described here, with

$$\hat{\Delta} s^{(i)}_p(m) = \hat{s}^{(i-1)}_p(m) - \hat{s}^{(I)}_{p+L_R}(m-1), \qquad (4)$$

taking advantage of the appropriate ΔL current decisions on symbols of block m (iteration $i-1$) and the final decisions regarding the last ΔL symbols of block $m-1$ (iteration I). Obviously, for $i = 1$ no previous decisions regarding block m are available; therefore, we should assume $\hat{s}^{(0)}_p(m) = 0$ when using (4) for $i = 1$. Certainly, there is a more efficient way of using the DDC aid: it consists of replacing the hard decisions by some kind of soft decisions, derived from the soft information that a SISO (Soft-In, Soft-Out) channel decoder can provide. An appropriate choice is a Soft Decision Directed Correction (SDDC) scheme, as an alternative to the DDC scheme described above, where the $\hat{\boldsymbol{\Delta}}^{(i)}(m)$ vector is replaced by

$$\overline{\boldsymbol{\Delta}}^{(i)}(m) = [\underbrace{0\ldots0}_{N-L},\underbrace{\overline{\Delta s}^{(i)}_{N-L}(m)\ldots\overline{\Delta s}^{(i)}_{N-L_R-1}(m)}_{\Delta L},\underbrace{0\ldots0}_{L_R}]^T. \quad (5)$$

In this new vector, for $p = N - L, \ldots, N - L_R - 1$,

$$\overline{\Delta s}^{(i)}_p(m) = \overline{s}^{(i)}_p(m) - \overline{s}^{(I)}_{p+L_R}(m-1), \qquad (6)$$

with $\overline{s}^{(i)}_p(m)$ and $\overline{s}^{(I)}_{p+L_R}(m-1)$ being mean symbol values (in the statistical sense) rather than hard decisions. In Appendix A, we indicate a way to compute these values, based on the LLRs of the coded bits, provided by the channel decoder, for a QPSK (Quaternary Phase Shift Keying) modulation. It should be noted that, in this case,

$$\overline{s}^{(i-1)}_p(m) = \frac{\sigma_s}{\sqrt{2}} \cdot \left(\tanh\left(\frac{L^{(i-1)}_{p,I}(m)}{2}\right) + j\tanh\left(\frac{L^{(i-1)}_{p,Q}(m)}{2}\right)\right), \quad (7)$$

where $L^{(i-1)}_{p,I}(m)$ and $L^{(i-1)}_{p,Q}(m)$ are the LLRs of the "in-phase coded bit" and "quadrature coded bit", respectively. Of course, the mean symbol value can be expressed as a function of the I/Q correlation coefficients $(\rho^{(i-1)}_{p,I}(m), \rho^{(i-1)}_{p,Q}(m))$ and the I/Q "decisions" $(\hat{s}^{(i-1)}_{p,I}(m), \hat{s}^{(i-1)}_{p,Q}(m))$: $\overline{s}^{(i-1)}_p(m) = \rho^{(i-1)}_{p,I}(m)\hat{s}^{(i-1)}_{p,I}(m) + j\rho^{(i-1)}_{p,Q}(m)\hat{s}^{(i-1)}_{p,Q}(m)$ (and similarly for $\overline{s}^{(I)}_p(m-1)$), where $0 \leq \rho^{(i-1)}_{p,I}(m) \leq 1$ and $0 \leq \rho^{(i-1)}_{p,Q}(m) \leq 1$. For $i = 1$, $\rho^{(i-1)}_{p,I}(m) = \rho^{(i-1)}_{p,Q}(m) = 0$, hence $\overline{s}^{(i-1)}_p(m) = 0$; after some iterations and/or when the SNR is high, typically $\rho^{(i-1)}_{p,I}(m) \approx 1$ and $\rho^{(i-1)}_{p,Q}(m) \approx 1$, leading to $\overline{s}^{(i-1)}_p(m) \approx \hat{s}^{(i-1)}_p(m)$.

When the proposed SDDC scheme is employed as an aid to a conventional, linear FDE, in an iterative way, the appropriate receiver configuration is as depicted in Fig. 1. By assuming the MMSE criterion, the entries of $\mathbf{F}(m)$ can be expressed as $F_k(m) = K_F(m)\hat{H}^*_k / \left(\hat{\alpha} + |\hat{H}_k|^2\right), k = 0, 1, \ldots, N-1$ when $\hat{\mathbf{H}} = [\hat{H}_0, \hat{H}_1, \ldots, \hat{H}_{N-1}]^T$ corresponds to the channel frequency response and $\alpha = \sigma^2_n/\sigma^2_s$, with σ^2_n denoting the noise variance. $K_F(m)$ can be adopted as a normalization factor (e.g., so that $\frac{1}{N}\sum_{k=0}^{N-1} F_k(m)\hat{H}_k = 1$). In Fig. 1, the "$\boldsymbol{\Delta}$ unit" computes $\overline{\boldsymbol{\Delta}}^{(i)}(m) = \overline{\mathbf{s}}^{(i)}(m) - \overline{\mathbf{s}}^{(I)}(m-1)$, with entries according to (5), Π and Π^{-1}

stand for "interleaver" and "deinterleaver", respectively, and $\odot$ denotes "element-by-element multiplication".

The soft demapper in Fig. 1 provides the inputs to the SISO decoder (LLRs of the several coded bits). The decoder outputs must correspond to the full soft information, not the extrinsic one (this is also true for the receivers of Figs. 2 and 3).

A simplified version of this receiver can be adopted, where the decoding procedures are removed from the iterative process. In this case, Fig. 1 may still be considered for receiver characterization, but with "decoder" outputs identical to the corresponding inputs, since there is no extrinsic information at all. It should also be mentioned that this SDDC-FDE technique is very similar to an iterative FDE technique proposed in [10], inspired in the RISIC (Residual ISI Cancellation) iterative algorithm proposed in [9] for OFDM receivers (see also related work in [11]).

An improved FDE receiver technique, capable of fully exploiting the soft information provided by the SISO decoder, is presented in the second part of next section, for reduced-CP, SC-based block transmission.

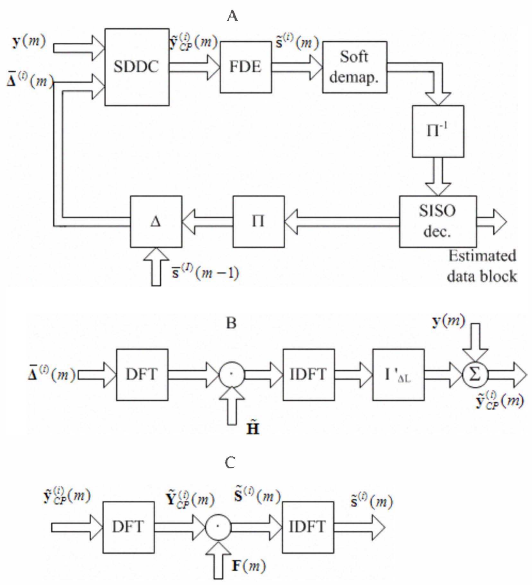

Fig. 1. SDDC-FDE receiver structure (A), with characterization of the SDDC unit (B) and the FDE unit (C).

3 Improved Turbo FDE Technique for Reduced-CP Transmission

3.1 New Look into Turbo FDE Techniques

When a full-length CP is employed, Turbo FDE techniques such as that proposed in [2], [3] can provide a strongly improved FDE performance, while avoiding a high complexity of implementation. Such techniques, which resort to a conventional, linear, single-tap FDE scheme, take advantage of the SISO decoder outputs in order to carry out a turbo soft-cancellation of residual ISI through the use of the soft information on the coded bits. According to the "switched APPLE/MF approach" (APProximate Linear (MMSE) Equalization/Matched Filtering) proposed in [2], [3], the FDE coefficients for each iteration are selected on the basis of an "average SNR" concerning the length-N block of equalizer outputs. This block plays the role of an extrinsic information on the coded data block, delivered by the equalizer to the channel SISO decoder. For each iteration, the receiver uses the algorithm (APPLE (MMSE) or MF) leading to the largest estimated SNR.

In the following, we describe a new "conventional" (for full-length CP conditions) Turbo FDE technique which is strongly related to that proposed in [2], [3], but replaces the selection principle (APPLE/MF) regarding the linear FDE parameters by an appropriate "compromise choice": in fact, the values of the linear FDE parameters are adaptively adjusted, iteration by iteration, according to the available block of SISO decoder outputs. The proposed receiver structure is as depicted in Fig. 2. At the equalizer output, the time-domain vector $\tilde{\mathbf{s}}^{(i)}(m)$ is the IDFT of

$$\tilde{\mathbf{S}}^{(i)}(m) = \mathbf{F}^{(i)}(m) \odot \mathbf{Y}(m) + \mathbf{G}^{(i)}(m). \quad (8)$$

Therefore, the N entries of $\tilde{\mathbf{S}}^{(i)}(m)$ can be written as

$$\tilde{S}_k^{(i)}(m) = F_k^{(i)}(m) Y_k(m) + G_k^{(i)}(m), \quad (9)$$

where $F_k^{(i)}(m), k = 0, 1, \ldots, N-1$, are the multiplicative FDE parameters for iteration i, and

$$G_k^{(i)}(m) = (\gamma^{(i)}(m) - F_k^{(i)}(m)\hat{H}_k)\overline{S}_k^{(i-1)}(m), \quad (10)$$

with $\gamma^{(i)}(m) = (1/N)\sum_{k=0}^{N-1} F_k^{(i)}(m)\hat{H}_k$ are complementary FDE parameters for ISI soft-cancelation purposes. $\overline{\mathbf{S}}^{(i-1)}(m) = [\overline{S}_0^{(i-1)}(m), \overline{S}_1^{(i-1)}(m), \ldots, \overline{S}_{N-1}^{(i-1)}(m)]^T$ is the DFT of $\overline{\mathbf{s}}^{(i-1)}(m) = [\overline{s}_0^{(i-1)}(m), \overline{s}_1^{(i-1)}(m), \ldots, \overline{s}_{N-1}^{(i-1)}(m)]^T$, resulting from the soft information provided by the SISO decoder as shown in Appendix A, when assuming a QPSK modulation. As to the multiplicative FDE parameters, alternatively to $F_k^{(i)}(m) = \hat{H}_k^*/(\hat{\alpha}+|\hat{H}_k|^2)$ (APPLE) or $F_k^{(i)}(m) = \hat{H}_k^*$ (MF) [2], [3], we adopt

$$F_k^{(i)}(m) = \frac{K_F^{(i)}(m)\hat{H}_k^*}{\hat{\alpha} + (1-(\hat{\rho}^{(i-1)}(m))^2)|\hat{H}_k|^2}, \quad (11)$$

where $\hat{\rho}^{(i-1)}(m) = \frac{1}{N}\sum_{n=0}^{N-1} \frac{E[s_n^*(m)\hat{s}_n^{(i-1)}(m)]}{E[|s_n(m)|^2]}$ is an "overall correlation coefficient". It can be obtained as an average value of the $2N$ correlation coefficients per bit $(\rho_{n,I}^{i-1}(m), \rho_{n,Q}^{i-1}(m))$, derived from the SISO decoder outputs. $K_F^{(i)}(m)$ is a normalization factor: using $\gamma^{(i)}(m) = 1$, $\tilde{s}_n^{(i)}(m) = s_n(m) + \xi_n^{(i)}(m)$, where $\xi_n^{(i)}(m)$ is the zero-mean "error" (assumed to be approximately complex Gaussian) concerning symbol $s_n(m)$ at the FDE output. Under the "Gaussian assumption", the LLRs of the "in-phase bit" and the "quadrature bit", at the SISO decoder input, are given by $L_{n,I}^{(i)}(m) = \frac{\sqrt{8}}{\sigma_{eq}^{2(i)}(m)}\sigma_s Re\{\tilde{s}_n^{(i)}(m)\}$ and $L_{n,Q}^{(i)}(m) = \frac{\sqrt{8}}{\sigma_{eq}^{2(i)}(m)}\sigma_s Im\{\tilde{s}_n^{(i)}(m)\}$ respectively, where $\sigma_{eq}^{2(i)}(m)$ is the mean-squared error in the time-domain samples $\tilde{s}_n^{(i)}(m)$. It can easily be estimated as $\hat{\sigma}_{eq}^{(i)}(m) = \frac{1}{N}\sum_{n=0}^{N-1}\left|\tilde{s}_n^{(i)}(m) - s_n^{'(i)}(m)\right|^2$, with $s_n^{'(i)}(m) = \frac{\sigma_s}{\sqrt{2}}\left(\text{sgn}(Re\{\tilde{s}_n^{(i)}(m)\}) + j\text{sgn}(Im\{\tilde{s}_n^{(i)}(m)\})\right)$.

We point out that, for $i = 1$, the $F_k^{(i)}(m)$ parameters meet the MMSE criterion, since $\hat{\rho}^{(i-1)}(m) = 0$ in (11). After a number of iterations and/or for high SNRs, typically $\hat{\rho}_{n,I}^{(i-1)}(m) \approx 1$ and $\hat{\rho}_{n,Q}^{(i-1)}(m) \approx 1$, leading to $\hat{\rho}^{(i-1)}(m) \approx 1$ in (11), and, therefore, to $F_k^{(i)}(m)$ parameters approximately in accordance with the MF criterion. It should also be noted that the proposed multiplicative FDE coefficients can be written as $F_k^{(i)}(m) = K_F^{'(i)}(m)\hat{H}_k^*/(\hat{\alpha}^{'(i)}(m) + |\hat{H}_k|^2)$, where $K_F^{'(i)}(m) = K_F^{(i)}(m)/(1-(\hat{\rho}^{(i-1)}(m))^2)$ and $\hat{\alpha}^{'(i)}(m) = \hat{\sigma}_n^2/\hat{\sigma}_s^{2(i)}(m)$, with $\hat{\sigma}_s^{2(i)}(m) = \sigma_s^2(1-(\hat{\rho}^{(i-1)}(m))^2)$. Having in mind the symbol variance parameter given by (15), the proposed $F_k^{(i)}(m)$ coefficients can be regarded as roughly approximating the MMSE criterion for all iterations, taking into account the available information from the SISO decoder.

To conclude, let us consider a simplified iterative FDE implementation, based on the ideas above, where no decoding effort is really involved in the FDE process. The corresponding receiver derives from that shown in Fig. 2, by suppressing the SISO decoder; therefore, no extrinsic information is provided to help the iterative FDE process, and the soft FDE outputs in a given iteration $(i-1)$ are directly used to compute $F_k^{(i)}(m)$ and $G_k^{(i)}(m)$ for next iteration. An additional simplification is to replace the correlation coefficient concerning every bit by $\hat{\rho}^{(i-1)}(m)$ in the computation of $\overline{s}_n^{(i-1)}(m)$, leading to $\overline{S}_k^{(i-1)}(m) = \hat{\rho}^{(i-1)}(m)\hat{S}_k^{(i-1)}(m)$, $([\hat{S}_0^{(i-1)}(m), \hat{S}_0^{(i-1)}(m), \ldots, \hat{S}_{N-1}^{(i-1)}(m)]^T$ is the DFT of $[\hat{s}_0^{(i-1)}(m), \hat{s}_0^{(i-1)}(m), \ldots, \hat{s}_{N-1}^{(i-1)}(m)]^T)$. Then,

$$\tilde{S}_k^{(i)}(m) = F_k^{(i)}(m)Y_k(m) - B_k^{(i)}(m)\hat{S}_k^{(i-1)}(m) \quad (12)$$

where

$$B_k^{(i)}(m) = \hat{\rho}^{(i-1)}(m)(F_k^{(i)}(m)\hat{H}_k - \gamma^{(i)}(m)), \quad (13)$$

with eqns. (11), (12) and (13) jointly defining an IB-DFE (Iterative Block Decision Feedback Equalization) receiver, as proposed in [4] and extended in [5]-[7]. It should be noted that, for implementation of an IB-DFE receiver, one can simply use $\hat{\rho}^{(i)}(m) = 1 - 2\hat{P}_e^{(i)}(m)$, with $\hat{P}_e^{(i)}(m) \approx Q(\sqrt{\sigma_s^2/\hat{\sigma}_{eq}^{(i)2}(m)})$, where $Q(\cdot)$ denotes the Gaussian error function.

3.2 Generalized SDDC-Aided Turbo FDE

When the CP length (L_R) is smaller than the channel memory order (L), good performances through the

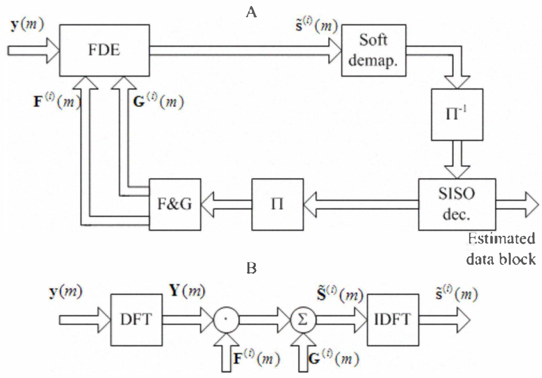

Fig. 2. Turbo FDE receiver structure (A) and characterization of the FDE unit (B).

iterative receiver technique of Fig. 2 cannot be ensured. In this case, Fig. 3 shows a suitable receiver technique, which somehow combines the capabilities of the techniques in Figs. 1 and 2. This technique actually uses an SDDC aid, as proposed in Sec. 2, to the Turbo FDE technique described in Sec. 3.1. This means that, for each iteration of the Turbo FDE scheme, the time-domain channel input to the FDE unit is updated, not only the FDE parameters. Of course, as in Sec. 3.1, one may consider to suppress the contribution of SISO decoding, in the context of Fig. 3, for the sake of implementation simplicity.

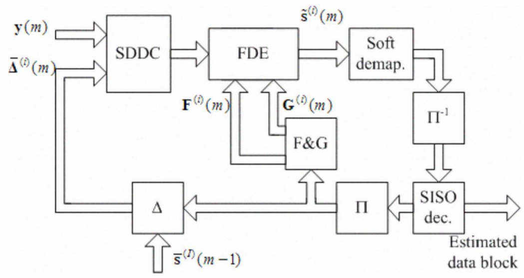

Fig. 3. Turbo SDDC-FDE receiver structure.

4 Performance Results

A set of numerical results is presented below, with regards to broadband transmission over a strongly frequency-selective Rayleigh fading channel, when the iterative techniques of Secs. 2 and 3 are employed, under perfect channel estimation. We adopt the power delay profile type C within HIPERLAN/2, with uncorrelated Rayleigh fading on the different paths. A conventional CP-assisted block transmission scheme is assumed, with $N = 256$ QPSK data symbols per block. The duration of the useful part of each block is 5μs and we consider either a full-length CP $(L_R = L = 64)$ or a reduced CP $(L_R = L/8 = 8)$; when considering the latter alternative, about 1/3 of the channel impulse response energy falls outside the time interval of the reduced CP.

Figs. 4 and 5 are concerned to simplified iterative FDE techniques with no channel decoding involved. Fig. 4 shows the uncoded BER for both the SDDC-FDE receiver of Sec. 2 and the Turbo FDE receiver of Sec. 3.1, when $L_R = L/8 = 8$): clearly, the Turbo FDE performance is seriously affected by the shortened CP, exhibiting an error floor at about 3.5×10^{-3}; the receiver technique of Sec. 2 , which has a worse performance for low SNR, achieves an improved performance for high SNR, obviously due to the soft cancellation of the reduced-CP effects. The uncoded BER results of Fig. 5 for $L_R = L/8 = 8$ correspond to the SDDC-aided techniques of Secs. 2 and 3.2 (Turbo FDE results for $L_R = L = 64$ are also shown): the benefits of jointly using a Turbo FDE approach and an SDDC aid are evident, allowing BER values below 10^{-3}; however, the lack of a SISO decoding contribution for the iterative FDE process still is a serious limitation.

For the performance results of Figs. 6, 7 and 8, we assumed a rate-1/2 convolutional code with $G(D) = [1 \quad (1 + D^2)/(1 + D + D^2)]$, a low-complexity SISO decoding through the use of the Max-Log-MAP algorithm [12], and the close equalization/decoding cooperation which is allowed by the receiver structures of Figs. 1, 2 and 3, as described in Secs. 2 and 3. Fig. 6 shows coded BER performances under full-length CP conditions ($L_R = L$) for both the Turbo FDE technique of Sec. 3.1 and the Turbo FDE technique which uses the APPLE/MF approach [2], [3]: very similar performances are achieved, close to the appropriate MF bound performance (for the coded SC transmission under consideration, with $L_R = L = 64$ and $N = 256$), with a small advantage for the Turbo FDE technique proposed in this paper. Fig. 7 shows coded BER performances when $L_R = L/8 = 8$, for both the Turbo FDE technique of Sec. 3.1 and the Turbo SDDC-FDE technique of Sec. 3.2: the advantage of the SDDC-aided technique is very clear, and it should be noted that it allows a close approximation to the new MF bound (with $L_R = L/8 = 8$ and $N = 256$, for the coded SC transmission under consideration); on the other hand, the conventional Turbo FDE technique (designed for full-length CP conditions) is not able to avoid a significant performance degradation, characterized by a clear error floor. To conclude, Fig. 8 compares coded BER results in the following cases: the SDDC-FDE technique of Sec. 2, with $L_R = L/8 = 8$, the Turbo FDE technique of Sec. 3.1, with $L_R = L = 64$, and the Turbo SDDC-FDE technique of Sec. 3.2, with $L_R = L/8 = 8$. A special attention should be paid to the following pair of performance curves: the best solid line, as compared to the best dashed line (Turbo FDE performances). We must keep in mind that, when reducing the CP length from $L_R = L = 64$ to $L_R = L/8 = 8$ (e.g., to increase the bandwidth efficiency by about $\left(\frac{L-L_R}{N+L_R}\right) \times 100\%$, which gives 21.2% in our case), the maximum achievable power efficiency gain is $10\log_{10}((N + L)/(N + L_R)) \approx 0.84$dB. Therefore, it is clear that the SDDC-aided approach can practically ensure that there is no degradation of the power efficiency as a downside of that reduction, and, on the contrary, there is a gain close to the maximum.

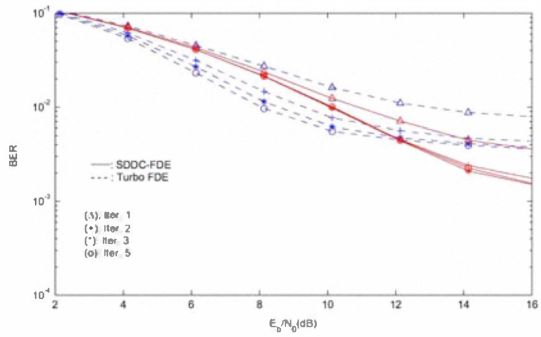

Fig. 4. Uncoded BER performances for $L_R < L$, when using the techniques of Sec. 2 and Sec. 3.1.

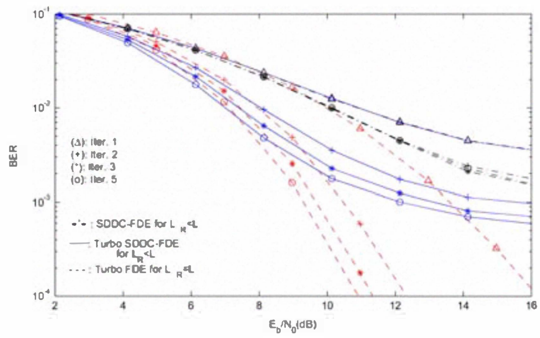

Fig. 5. Uncoded BER performances for the SDDC-aided techniques ($L_R < L$) and the Turbo FDE technique of Sec. 3.1 ($L_R = L$).

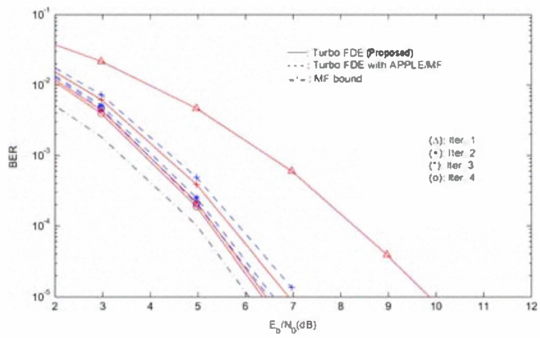

Fig. 6. Coded BER performances when $L_R = L$, for the Turbo FDE technique proposed in Sec. 3.1 and the Turbo FDE technique which uses the APPLE/MF approach [2], [3] (The MF bound is also included).

5 Conclusions

The advantages of the SDDC-aided, FDE and Turbo FDE techniques were emphasized, namely the fact that

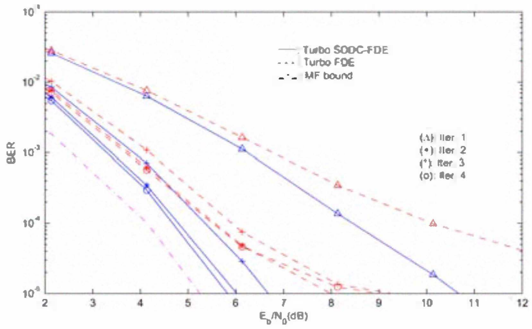

Fig. 7. Coded BER performances when $L_R < L$, showing the advantages of the SDDC-aided Turbo FDE technique over the Turbo FDE technique of Sec. 3.1

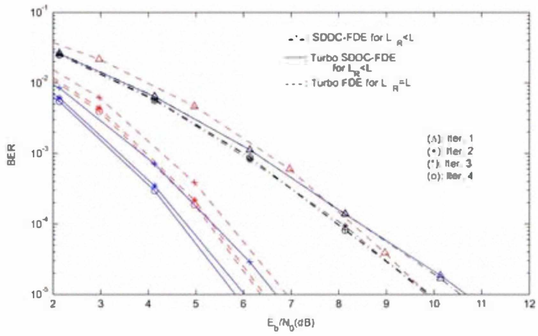

Fig. 8. Coded BER performances for the SDDC-aided techniques ($L_R < L$) and the Turbo FDE technique of Sec. 3.1 ($L_R = L$).

they can operate satisfactorily even when, for the sake of implementation simplicity, no decoding operations are used to improve the iterative FDE process. Moreover, they can achieve the maximum power efficiency gains that a strong CP reduction allows.

To conclude, we point out that Turbo FDE techniques, such as that described in Sec. 3.1, designed for full-length CP conditions, exhibit a strong "complexity advantage" over turbo equalization techniques in the time domain [3], but suffer from the "power/bandwidth drawback" which is inherent to the full-length CP. By allowing a reduced-CP system choice, the proposed Turbo SDDC-FDE technique essentially preserves the above-mentioned advantage, while avoiding the above-mentioned drawback. This turbo technique, (as proposed in sec. 3.2), also has a strong "performance/complexity advantage" over the techniques of secs 2 and 3.1, and seems to be especially recommendable for the SC-based uplink of future broadband systems, following ideas introduced in [13].

Appendix A. QPSK Symbol Statistics Using Soft Decoder Outputs

Let us assume QPSK symbol coefficients $s_n = s_{n,I} + js_{n,Q}$, with $s_{n,I} = \pm\sigma_s/\sqrt{2}$ and $s_{n,Q} = \pm\sigma_s/\sqrt{2}$ ($n = 0, 1, \ldots, N-1$), according to the coded data block. When the LLRs (log-likelihood ratios) concerning the nth in-phase bit and the nth quadrature bit, as provided by the channel decoder, are $L_{n,I}$ and $L_{n,Q}$, respectively, the resulting expected value $\overline{s}_n$ can be expressed as $\overline{s}_n = \overline{s}_{n,I} + j\overline{s}_{n,Q}$ with $\overline{s}_{n,I} = \frac{\sigma_s}{\sqrt{2}} \tanh\left(\frac{L_{n,I}}{2}\right)$ and $\overline{s}_{n,Q} = \frac{\sigma_s}{\sqrt{2}} \tanh\left(\frac{L_{n,Q}}{2}\right)$.

Let us define the "coded bit decisions", $\hat{s}_{n,I} = \pm\sigma_s/\sqrt{2}$ and $\hat{s}_{n,Q} = \pm\sigma_s/\sqrt{2}$, according to the signs of $L_{n,I}$ and $L_{n,Q}$, respectively, and the following correlation coefficients: $\rho_{n,I} = \frac{E[s_{n,I}\hat{s}_{n,I}]}{E[|s_{n,I}|^2]}$; $\rho_{n,Q} = \frac{E[s_{n,Q}\hat{s}_{n,Q}]}{E[|s_{n,Q}|^2]}$. Since $\rho_{n,I} = 1 - 2\ \mathrm{Prob}(\hat{s}_{n,I} = -s_{n,I}|L_{n,I}) = \tanh(|L_{n,I}|/2)$ and $\rho_{n,Q} = 1 - 2\ \mathrm{Prob}(\hat{s}_{n,Q} = -s_{n,Q}|L_{n,Q}) = \tanh(|L_{n,Q}|/2)$ (leading to $0 \leq \rho_{n,I} \leq 1$ and $0 \leq \rho_{n,Q} \leq 1$), the average values $\overline{s}_{n,I}$ and $\overline{s}_{n,Q}$ can be written as follows:

$$\overline{s}_{n,I} = \rho_{n,I}\hat{s}_{n,I}; \quad \overline{s}_{n,Q} = \rho_{n,Q}\hat{s}_{n,Q} \tag{14}$$

Regarding the nth QPSK symbol, a variance parameter can also be derived from the pair of decoder outputs and expressed as a function of the I and Q correlation coefficients. By defining $\rho_n^2 = (\rho_{n,I}^2 + \rho_{n,Q}^2)/2$,

$$\sigma_{s_n}^2 = \sigma_s^2 - |\overline{s}_n|^2 = \sigma_s^2(1 - \rho_n^2). \tag{15}$$

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