

Space Time Block Codes with Turbo Codes

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Abstract

This paper aims to provide efficient solutions to improve the power efficiency, like implementation of schemes using MIMO (Multiple-Input Multiple-Output), with error correction codes. Thanks to the union of these two techniques is possible to obtain a low error probability with low power consumption. This paper shows STBC (Space Time Block Codes) with codes and diversity in the reception using a MRC (Maximum Ratio Combining). In this solution are studied Alamouti codes [1] and those proposed by Tarokh et al [2], with four and eight transmit antennas. Turbo Codes are used in the decoding algorithm with soft output, the log-MAP. The simulation results of the performance of error probability (BER) are performed in MATLAB with 4-QAM modulation in a Rayleigh channel.

Key Words: MIMO, STBC, Alamouti, Turbo Code, log-MAP

1. Introduction

The motivation for this work is the improvement of power efficiency to increase autonomy of mobile terminals or increase the quality of the communication.

In mobile communications, are mandatory services and applications with better quality, lower error probability and faster transmission rates. Since the transmission channel is responsible for the introduction of noise, interference and fading, the received information appears degraded, is necessary to use techniques to improve performance. One of the simple solutions for achieving this goal is to increase the transmission power in mobile terminals or increase the bandwidth. Although these solutions are valid, none of them are implemented, because the increase of the transmission power in mobile terminals would increase battery consumption and cost, besides being detrimental to our health. Increasing the bandwidth would be an easy way to solve performance problems and low transmission rates, however there is a price to pay, because the allocation of spectrum is limited and expensive.

The traditional mobile communications use systems with a transmitting antenna and one receiving antenna, designed for SISO (Single-Input-Single-Output), are not effective to combat fading. One solution to combat fading proposed in this paper is the use of MIMO techniques using STBC coding with diversity reception, using error correction codes (turbo codes). Fortunately with the combination of these two techniques is possible to obtain a low error probability with low power consumption, without increasing the bandwidth and transmit power in mobile terminals.

Beyond this introduction, is described in section 2 the STBC schemes used in the simulations and in section 3 is described the solution used for correction of errors (Turbo Codes). To prove that this solution proposed in this paper is achieved the simulations are analyzed in section 4 and then their conclusions are described in section 5.

2. STBC schemes

In 1998, Alamouti proposed a simple scheme of diversity [1], with two transmission antennas and one reception antenna (equation (2.1)). The extension of the Alamouti STBC for more than two transmit antennas was proposed later by Tarokh et al [2]. In this paper, in addition to Alamouti matrix, the matrix is presented with four and eight antennas transmission (equation (2.2) and equation (2.3)). STBC schemes consist in encoding symbols carried in space and time, which are then transferred as a matrix (Figure 2.1).

$$X_2^c = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}^T \quad (2.1)$$

$$X_4^c = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & -x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}^T \quad (2.2)$$

$$X_8^c = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 & x_1^* & -x_2^* & -x_3^* & -x_4^* & -x_5^* & -x_6^* & -x_7^* & -x_8^* \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 & x_2^* & x_1^* & -x_4^* & x_3^* & -x_6^* & x_5^* & x_8^* & -x_7^* \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 & x_3^* & x_4^* & x_1^* & -x_2^* & -x_7^* & -x_8^* & x_5^* & x_6^* \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 & x_4^* & -x_3^* & x_2^* & x_1^* & -x_8^* & x_7^* & -x_6^* & x_5^* \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 & x_5^* & x_6^* & x_7^* & x_8^* & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 & x_6^* & -x_5^* & x_8^* & -x_7^* & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 & x_7^* & -x_8^* & -x_5^* & x_6^* & x_3^* & -x_4^* & x_1^* & x_2^* \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & x_1 & x_8^* & x_7^* & -x_6^* & -x_5^* & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}^T \quad (2.3)$$

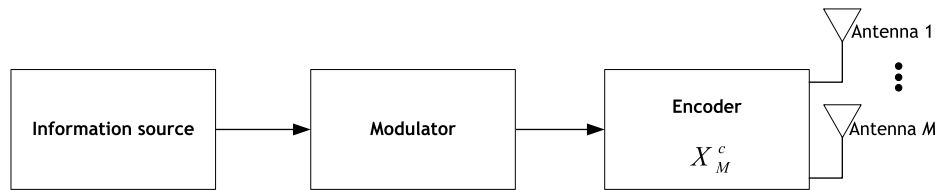


Figure 2.1 - Transmission scheme with STBC.

The rate of each matrix is obtained by dividing the number of rows (number of transmit antennas) with the number of columns (time instants) (equation (2.4)).

$$R = \frac{K}{T} \quad (\text{bit/s}) \quad (2.4)$$

Spectral efficiency is defined by [3]:

$$\varepsilon = R \log_2(C) \quad \text{bit/s/Hz} \quad (2.5)$$

where C is the number of modulation symbols used.

The received signal with the Alamouti scheme (Figure 2.2) can be defined by:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \times \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (2.6)$$

where y_1 and y_2 correspond to the bits received at two instants of time, h_1 e h_2 correspond to the Rayleigh channel between the two antennas with one receiver.

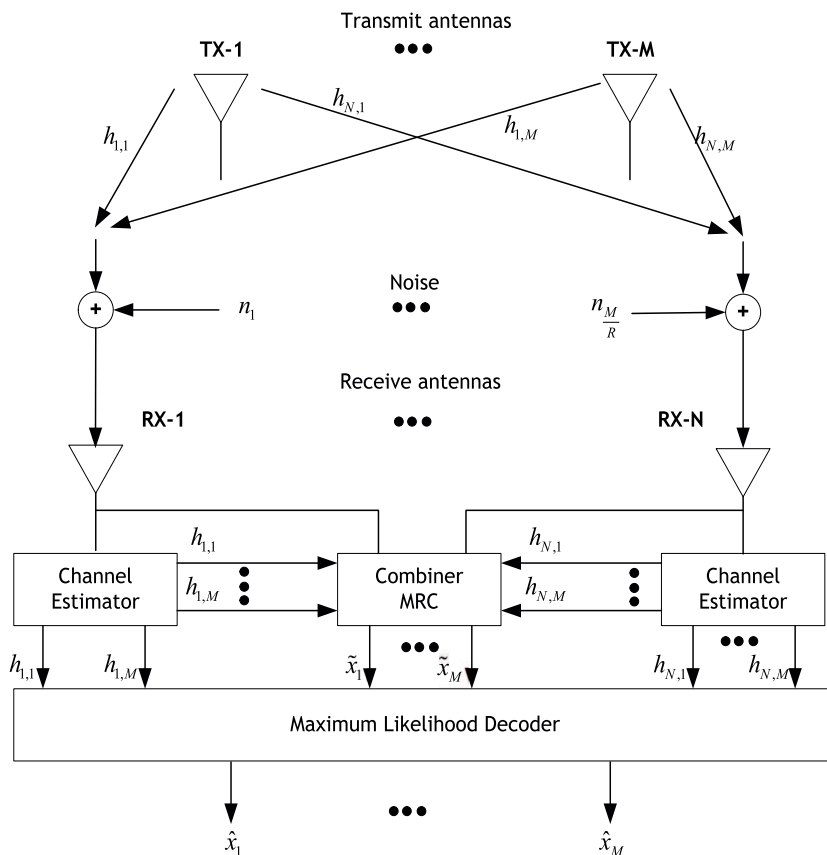


Figure 2.2 - Reception with the Alamouti scheme [3].

The variables n_1 and n_2 correspond to the noise sample in the respective moments of time. Equation (2.6) can be rewritten as follows:

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{H_2^c} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (2.7)$$

Generalizing equation (2.7) for more transmission antennas is:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{x+\frac{M}{R}}^* \\ \vdots \\ y_{\frac{M}{R}}^* \end{bmatrix} = H_M^c \times \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{\frac{M}{R}} \end{bmatrix} \quad (2.8)$$

where M is the number of antennas. The variable $x = 0$ with two transmit antennas (Alamouti) and $x = 1$ for more than two antennas. At the reception, the receiver has perfect knowledge of the channel so is used MRC and the maximum likelihood detection [3]. The MRC is shown in equation (2.9), which shows the symbols from the received signals and the estimation of channel. After the combination of two signals, these are then sent to a maximum likelihood decoder to estimate the transmitted symbols (equation (2.10)).

$$\begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_M \end{bmatrix} = (H_M^c)^{*T} \times \begin{bmatrix} y_1 \\ \vdots \\ y_{x+\frac{M}{R}}^* \\ \vdots \\ y_{\frac{M}{R}}^* \end{bmatrix} \quad (2.9)$$

$$\begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_M \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_M \end{bmatrix} \times \frac{1}{|H_M^c|^2} \quad (2.10)$$

3. Turbo Code

The simulation of STBC uses the TC (Turbo codes) [4,5] (Figure 3.1) with two RSCC (Recursive and Systematic Convolutional Coders), eight states, concatenated in parallel and separated by pseudo-random interleavers. The matrix is given by:

$$G(D) = \begin{bmatrix} 1 & 1+D+D^3 \\ 1+D^2+D^3 \end{bmatrix} \quad (3.1)$$

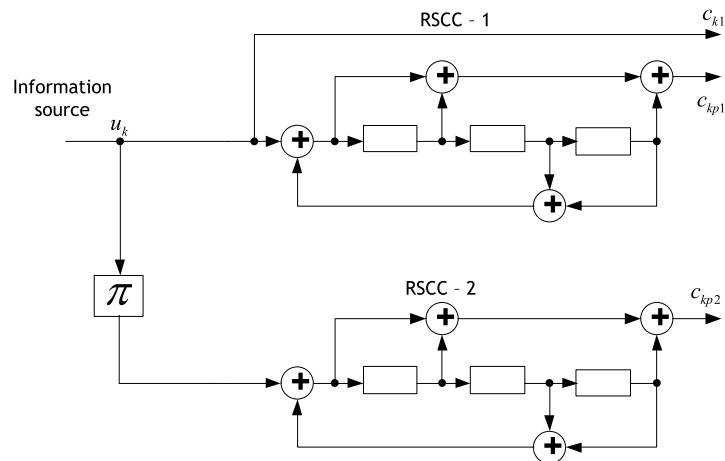


Figure 3.1 - Schematic block TC.

3.1 Iterative Decoding

Consider the diagram of Figure 3.2, which consists of two decoders RSCC concatenated and separated by a interleaver. The first decoder uses the received systematic bit sequence y_{k1} and received parity sequence y_{kp1} generated by the first encoder with the extrinsic information produced by the second decoder $E_1^{P'}(u_k)$ after deinterleaving. In the first iteration, the extrinsic information $E_1^{P'}(u_k)$ is zero. The output of log-MAP decoding, of the first decoder is a soft output, known as $L_1(u_k|y)$. The second decoder takes as input $y_{k1}^{(P)}$, which is obtained by the permutation of the sequence of bits y_{k1} and received parity sequence y_{kp2} generated by the second encoder with the extrinsic information produced by the second decoder $E_1^P(u_k)$. The $E_1^P(u_k)$ extrinsic information is obtained through the equation (3.2), with interleaver:

$$E_1(u_k) = L_1(u_k | y) - E_1^{P'}(u_k) - y_{kp1} \quad (3.2)$$

After a determined number of iterations, then the output of the second decoder $L_2(u_k|y)$ is subjected to the deinterleaver, resulting in $L(u_k|y)$ and by a decision maker is the sequence obtained \hat{u}_k estimated. In each iteration the extrinsic information is calculated $E_2(u_k)$, through the equation (3.3) which is subject to extrinsic information that originates the deinterleaved $E_1^{P'}(u_k)$, thus returning to the beginning of turbo decoding.

$$E_2(u_k) = L_2(u_k | y) - E_1^P(u_k) - y_{k1}^P \quad (3.3)$$

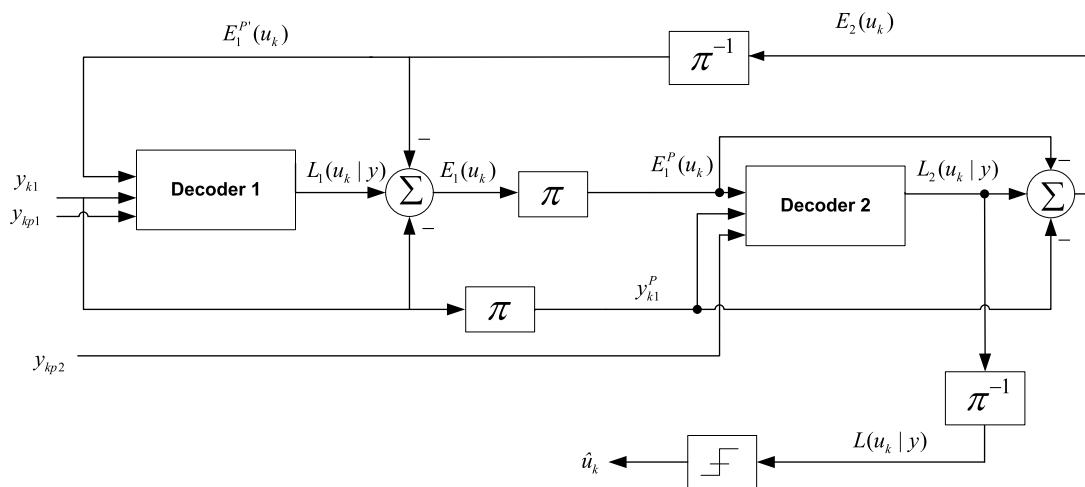


Figure 3.2 - Block diagram of a decoder for TC.

3.1.1 Iterative decoding based on the log-MAP algorithm

The log-MAP algorithm is equivalent to the MAP algorithm, with the multiplication of the MAP algorithm transformed into additions. In [6] three probabilities are calculated: $\Gamma_k(s', s)$, $A_k(s)$ and $B_{k-1}(s)$, which is then reached, the probability "a posteriori" $L(u_k | y)$, is defined by:

$$L(u_k | y) = \max_{R_1}^* [A_{k-1}(s') + \Gamma_k(s', s) + B_k(s)] - \max_{R_0}^* [A_{k-1}(s') + \Gamma_k(s', s) + B_k(s)] \quad (3.4)$$

4. Analysis of results

In this work two types of scenarios are simulated: schemas SISO, Alamouti and STBC without diversity (Figure 4.1) and STBC scheme with four transmit antennas and 1,2,3,4,5,6,7 and 8 receiving antennas (Figure 4.2). For the first scenario, it follows that with the introduction of satellite transmission gain is obtained after a certain number of transmit antennas decreases, tending to zero (Figure 4.5a), the same can be concluded with STBC schemes with diversity in reception (Figure 4.5b). The theoretical curves of the error probability for the two scenarios are defined as:

$$P_b = \frac{1}{2} \left(1 - \sqrt{\frac{1}{1 + \frac{M}{E_b/N_0}}} \right) \times \sum_{q=0}^{Q-1} \left\{ \frac{\prod_{i=0}^q |2i-1|}{q! 2^q} \left(1 + \frac{E_b}{N_0} \right)^{-q} \right\} \quad (4.1)$$

where $Q = M \times N$, M is the number of transmit antennas and N the number of receiving antennas.

With the simulation diversity STBC schemes with TC using log-MAP algorithm (Figure 4.3), the gain from increasing the number of transmit antennas is low (Figure 4.6a), compared with what that was obtained in Figure 4.5a where there is a significant gain with little transmission antennas, tending to zero after a certain number of transmission antennas. With STBC schemes with diversity using the technique of TC (Figure 4.4), a significant gain is obtained with the introduction of receiving antennas, tending to zero after a number of receiving antennas (Figure 4.6b).

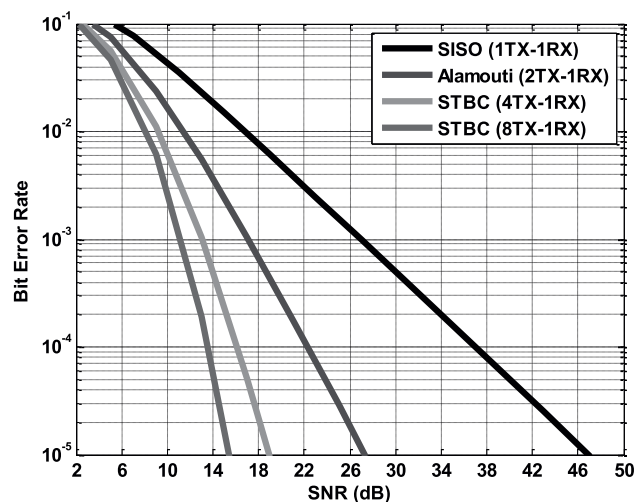


Figure 4.1 - Simulation schemes SISO, Alamouti STBC and without diversity reception.

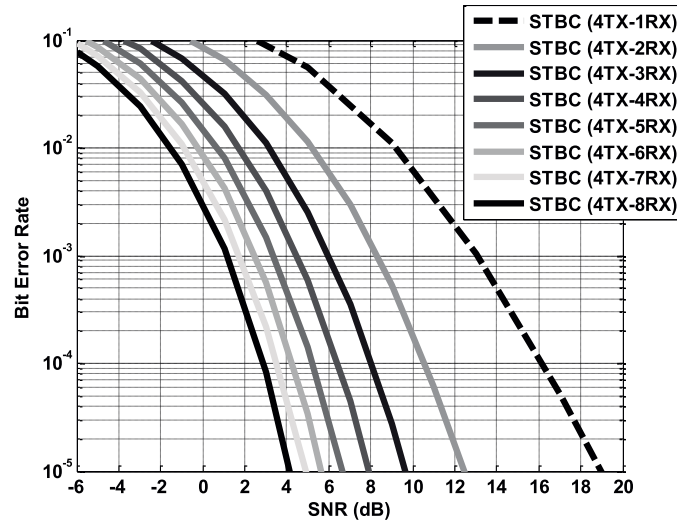


Figure 4.2 - Simulation STBC with four transmit antennas and diversity reception.

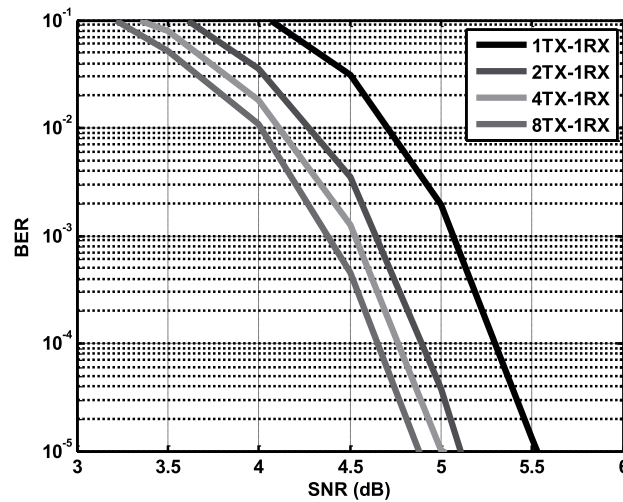


Figure 4.3 - Comparison schemes SISO, STBC without diversity and with TC (log-MAP), 10 iterations (rate=1/3, K=4).

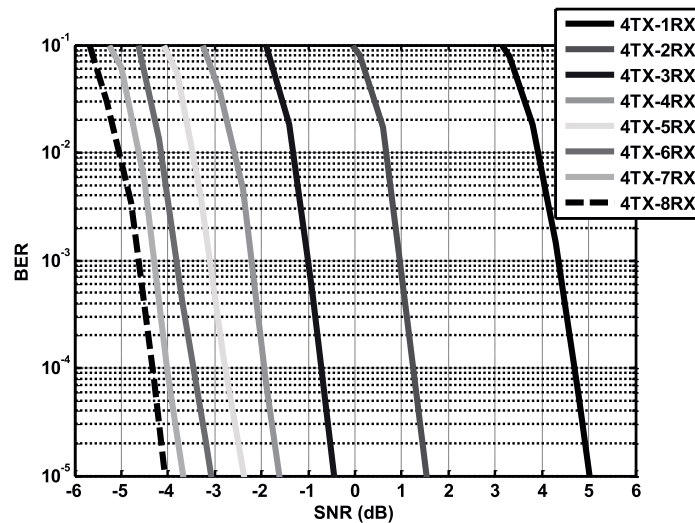


Figure 4.4 - Comparison with diversity STBC schemes with TC (log-MAP), 10th iteration (rate = 1/3, K = 4).

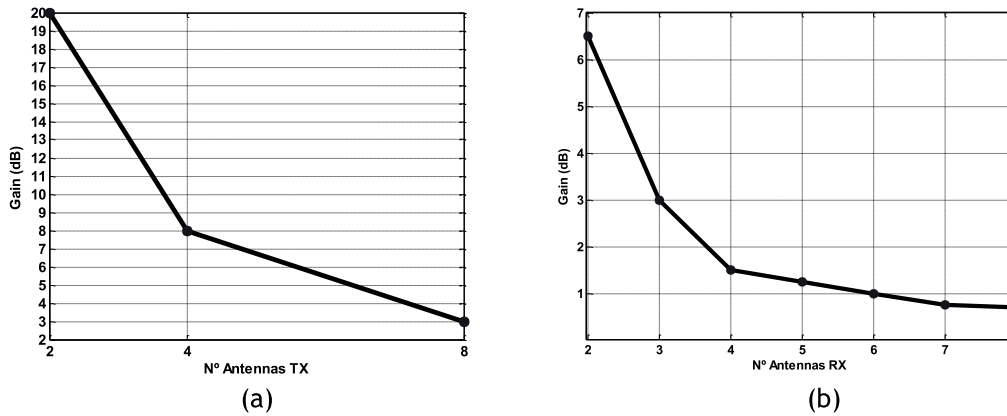


Figure 4.5 - Gain on STBC scheme with (a) 2, 4 and 8 transmit and one reception antenna (b) 4 transmission and reception antennas 2,3,4,5,6,7 and 8.

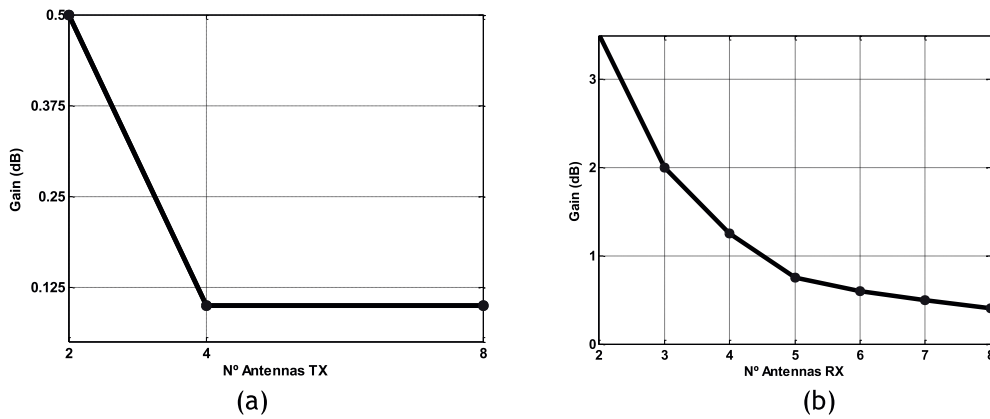


Figure 4.6 - Gain on STBC scheme, TC (log-MAP), 10th iteration (rate = 1/3, K = 4), with (a) 2, 4 and 8 transmission antennas and one reception antenna (b) 4 transmission antennas and 2,3,4,5,6,7 and 8 reception antennas.

Table 4.1 - Comparison of STBC without diversity in the reception with and without TC

Number of Transmitter antennas	Number of Receiver Antennas	
	1	
	Without TC	With TC
1	47	5.6
2	27	5.2
4	19	5
8	16	4.8

Table 4.2 - Comparison STBC scheme with four transmit antennas and reception 1,2,3,4,5,6,7 and 8, with and without TC.

Number of Transmitter antennas	Number of Receiver Antennas (Without TC)							
	1	2	3	4	5	6	7	8
4	19	12.5	9.5	8	6.75	5.75	5	4.25
	Number of Receiver Antennas (With TC)							
	5	1.5	-0.5	-1.75	-2.5	-3.1	-3.6	-4

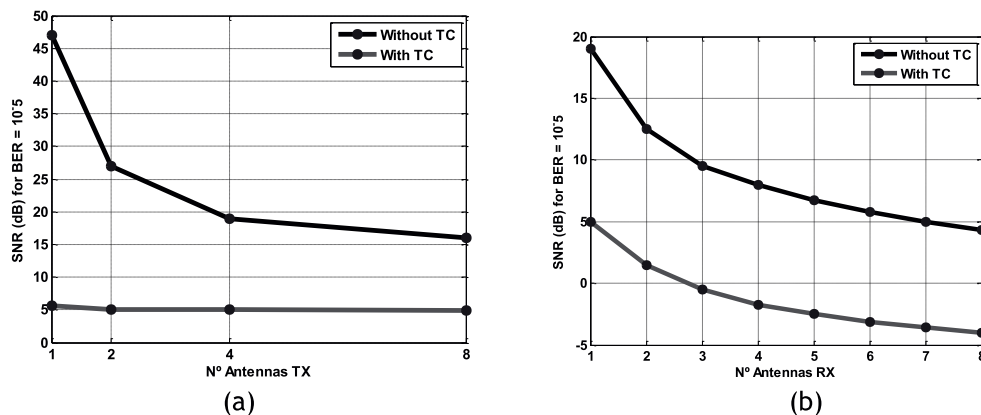


Figure 4.7- Comparison of the SNR required to obtain a BER of 10^{-5} for STBC schemes (a) 1,2,4 transmit antennas and 1 of reception (b) with 4 transmit antennas and 1,2,3,4,5,6,7,8 of reception, with and without TC.

5. Conclusions

Through the analysis of tables 5.1 and 5.2 and Figure 5.1, it follows that the two simulated scenarios, STBC without diversity and STBC with 4 transmit antennas and 1,2,3,4,5,6,7,8 reception antennas, with the introduction of TC it is possible to significantly reduce the SNR (Signal Noise ratio) compared to scenarios without TC. In other words, it is possible to achieve the same error probability with a lower SNR. However, the improvement on SNR with the increase of number of reception antennas decreases as long as the SNR decreases (see Figure 4.4). The STBC with TC and diversity in the reception, beyond the reduction of the SNR, a significant reduction in SNR is obtained with the addition of receiving antennas, until stabilizes.

The contribution of this paper is the achievement of great power efficiency thanks to the scheme STBC with TC. With this solution it's possible to achieve at same time a better quality of the communications and a reduction of power in the mobile terminals. In the future we will simulate with a higher constellation in QAM (e.g. 16-QAM) and other error correction codes.

6. References

1. Alamouti, S.M., "A Simple Transmit Diversity Technique for Wireless Communications", *IEEE Journal Selected Areas in Communications*, vol. 16, no.8, 1998, pp 1451-1458.
2. Tarokh, V., Jafarkhani, H., Calderbanh, A., "Space-Time Block Codes from Orthogonal Designs", *IEEE Transactions on Information Theory*, vol.45, no.5 1999, pp 1456-1467
3. Vucetic, Branka e Yuan, Jinhong, "Space-Time Coding", John Wiley & Sons, Ltd, 2003.
4. Berrou, C., Glavieux, A., Thitimajshima, P., "Near Shannon limit error-correcting coding and decoding: Turbo codes", *IEEE International Conference on Communications*, vol.2, 1993, pp 1064-1070.
5. S. Benedetto, D. Divsalar, G. Montorsi, F. Pollara, "Soft-Output Decoding Algorithms in Iterative Decoding of Turbo Codes", *The Telecommunications and Data Acquisition Progress Report 42-124*, pp. 63-87, 1996.
6. Abrantes, Sílvio, "Códigos Correctores de Erros em Comunicações Digitais", FEUP edições, 2010.